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Collective effects of bound electrons, free electrons and ions in a plasma

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bstract The collective behaviour of a plasma is investigated when the plasma is considered to be as mixture of compressible fluids of free ctrons, free ions, weakly bound electrons and neutral atoms A classical theory has been developed for understanding the dynamical behaviour of such plasma Non-relativistic closed system of fluid equations of a plasma is used which contain a population of bound electrons, the associated ions, the e electrons and free ions It is found that the Poynting theorem for energy conservation and the Maxwell stress elements have some new terms The rticle dynamical analysis and the energy exchange aspects of free electrons and bound electrons have also been considered Kinetic and kinematical pects of this model and other models have been discussed briefly

eywords Bound electrons, Maxwell stress element, energy flux

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Introduction

We formulate and discuss a non-relativistic, classical, closed systems of field equations of plasma regarded as a mixture of populations of bound electrons, free electrons, neutral particles and neutralising ions in presence of applied wave fields. Particle dynamics of bound electron in presence of applied wave fields, in the classical limit, exists and gives useful results in the Larmour precession effect, in the scattering theory of light of Rayleigh and Thomson, *etc*.

Including neutral particles, this plasma is a mixture of at least five compressible species of fluids, when the ion fluid for free electrons and bound electrons are distinguished. The force of simple harmonic motion, proportional to the field induced displacement of electrons about their ionic cores in addition to the Lorentz force, acts on the bound electrons only. Actually, the conservative central Coulomb potential of an atomic nucleus reduces to the centrifugal force of rotation of bound electrons about that nucleus. Its influence on polarization of the bound electrons is included in the electric displacement vector $\boldsymbol{D}$. So, it exists in the displacement current in the Ampere-Maxwell equation, which also contains the plasma current of the free charges. $\boldsymbol{D}$ moreover, appears in Gauss's theorem. Hence, the concept of the polarization vector for bound electrons is introduced phenomenologically in the Maxwell equations, and the Lorentz theory of electrodynamics obtains a closed system of guiding equations avoiding the empirical state relations of the phenomenological theory of electrodynamics [1]. Analytical definition of polarization, as the sum, over all species of charges of the product of charge density and field induced displacement, permits this type of mixing of the two classical theories.

This model of plasma though more difficult, is more realistic than the plasma of only free electrons and neutralising ions. In

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plasmas, some atoms remain neutral, but their valence electrons are weakly bound to their respective nuclei in partially ionized plasmas, for example, in plasmas in the ionosphere, in the cosmic spaces of the chromosphere, solar photosphere, and cool interstellar clouds, etc. Applicational possibility of this model also exists in laser plasma interaction and in wave interaction with solid state plasmas.

In sodium and other alkali metals, the valence electrons are weakly bound and the electron orbits are distorted by incident fields. So, considering these quasi free charges as harmonically bound to their respective nuclei, and ignoring the anharmonicity, from interaction of weak Coulomb field of atomic cores and other electrons, the optical properties of atoms excited by strong electromagnetic radiation, are determined [1].

Ignoring their collective effects, but assuming the existence of bound electrons in the background of free electron plasma, some non linear effects were studies earlier. These include the non-linearly induced precessional rotation of an elliptically polarized wave and the inverse Faraday effect (IFE) [2-6]. It is to be mentioned that previous authors did not consider the collective effects of bound electrons, free electrons and free ions for their study on the propagation of waves in the plasma.

In the present paper, the emphasis is on the formulation and establishment of the basic dynamics of the plasma, regarded as a mixture of several species of fluids, including the formulation of the self-consistent system of the basic equations, in the fluid approximation. It also contains a development of the Maxwell stress elements and of the Poynting theorem for energy flux and energy density of electro-magnetic field and the electro-acoustic field of compressibility, due to the collective effect of all the plasma constituents. The net energy flux is the sum of the electromagnetic radiation, and the flux of other energies, like the potential energy and the kinetic energy which evolve from interaction of the Lorentz force with correlated movement of the plasma constituents.

2. The existing theories of bound electrons in a plasma

Analysis of field induced motion of free electrons and bound electrons show how different and difficult their solutions and their interpretations are, compared to those for free electrons. Only the collective effects of free electrons are taken account of in the existing fluid and kinetic models for the development of the dynamics of plasmas. The collective effects of bound electrons are normally ignored. However, the particle dynamics exists for both free electrons and bound electrons in presence of EM waves and when the average mean free path is much greater than the dimensions of the plasma particles of the largest size. We consider these aspects in Subsections 2(a) and 2(b) for completeness of the report and for a parallel projection of their contrasts. The calculations for free electrons are extended upto the third order of small quantities, for the first harmonic of the field. The bound electron dynamics being much complicated, has not been extended to that extent.

2. (a) Particle dynamics of bound electrons in a plasma

The equation of electron motion in presence of the conservative potential of a Coulomb field towards the central nucleus $\phi(r)\left(=-e^2/r\right)$ and the Lorentz force, is

$$m\dot{u} = -eE - \frac{e}{c}(u \times B) - \nabla\phi, \qquad (1)$$

where dot denote the time derivative and u is the field-induced velocity.

For the electro magnetic radiation field from distance sources the force $-e(u \times B)/c$ is negligible compared to $-eE$, though $e(u \times B_0)/c$, from static applied magnetic field (B_0), is not necessarily negligible. And since

$$\nabla\phi : \frac{e^2}{r^3}\xi = \frac{e^2}{r_0^3}\xi$$

for circular orbits of electrons at constant radius r_0, Kepler third law of motion gives $\nabla\phi = m\omega_0^2\xi$ where $\omega_0^2 = e^2/mr_0^3$, ω_0 being the classical orbital electron frequency. It is a simple harmonic oscillator motion effect. The ratio

$$\frac{|\nabla\phi|}{|f_r|} = \frac{(e^2/r_0^3)\xi}{\ddot{\xi}(2e^2/3c^3)} = \frac{3c^3}{2r_0^3\omega^3}$$

between $\nabla\phi$ and the radiation damping force

$$f_r = \frac{2e^2}{3c^2}\ddot{\xi} = -\gamma u,$$

where $\gamma = (2e^2/3c^3)\omega^2$, is very small. At optical frequencies for instance, this ratio is about 10^7, $\omega_0 \simeq 10^{15}$/sec and $\gamma \simeq 10^8$/sec. Adding the small field of radiation damping, the non-conservative equation for electron motion, driven by applied electric field E is

$$\dot{u} = -\frac{e}{m}E - \frac{e}{mc}(u \times B_0) - \omega_0^2\xi + \gamma u.$$

Eq. (5) is not valid for bound electrons of high atomic number. The resonant frequency ω_0 of a radiative interaction is determined by the energy difference between the energy levels

$$\hbar\omega_0 = \Delta\varepsilon.$$

This ω_0 is not identified with the ω_0 of eq. (5) for the classical electron orbital frequency, and for interaction with powerful radiation fields including that on real atoms which have discrete energy levels and an elevated ground level that does not decay. However in the asymptotic limit of high quantum numbers n_0, when there is a transition for which the quantum number changes by one step $\Delta n = 1$, the radiation frequency becomes equal to the orbital frequency [7]. So, agreement is possible between these classical and quantum formalisms. Moreover, the classical oscillator equation generates the results of the quantum theory for absorption and scattering of polarization radiation even for small quantum numbers when ω_0 equals the transition frequency $\Delta\varepsilon / \hbar$ rather than the orbital frequency. The classical theory is valid for the polarization effects of normal Zeeman splitting with a correction factor for the electron spin, anomalous Zeeman splitting pattterns, *etc.* It describes well, phenomenologically, other atomic system of practical interest.

2(b) Motion of free electrons in presence of electro-magnetic wave

The nonlinear equations of motion of a single electron, between two successive collisions in a plasma, in presence of an electro-magnetic wave, linearly polarized in the ZX plane, are

$$m\dot{u}_x = -eE_x + \frac{e}{c}u_z H_y - \frac{m}{\tau}u_x , \tag{7}$$

$$m\dot{u}_y = \frac{m}{\tau}u_y , \tag{8}$$

$$m\dot{u}_z = -eE_z - \frac{e}{c}u_x H_y - \frac{m}{\tau}u_z , \tag{9}$$

where τ is the phenomenological collision time. The applied wave field has the stationary wave train form :

$$E_x = E\exp(i\theta) + \bar{E}\exp(-i\theta),\ \theta = (kz - \omega t) . \tag{10}$$

The collision frequency $\gamma (= 1/\tau)$ describes the damping of the motion in a statistical sense and insures a steady state response independent of the initial conditions. Using Faraday's law of induction in the linearized approximation, we get

$$H_y = \frac{kc}{\omega} E_x , \tag{11}$$

$$\xi_x = \frac{e}{m} E \frac{\exp(i\theta)}{\omega(\omega + i\gamma)} + c.c. , \tag{12}$$

$$j_x = -N_0 e\dot{\xi}_x = \frac{iN_0 e^2 E\exp(i\theta)}{m(\omega + i\gamma)} + c.c., \tag{13}$$

$$P_x = \chi^{(1)}(\omega) E_x = -N_0 e E_x , \tag{14}$$

$$\chi^{(1)}(\omega) = -\frac{N_0 e^2}{m\omega(\omega + i\gamma)} \tag{15}$$

where j_x is the wave induced current, N_0 is the average density of electrons in the plasma, P_x is the polarization (electric dipole moment per unit volume) and $\chi^{(1)}(\omega)$ is the electrical susceptibility of the plasma. At optical frequencies, since $\omega\tau >> 1$, the plasma dielectric constant

$$\varepsilon = 1 + 4\pi\chi^{(1)}(\omega) = 1 - \left(\omega_p^2 / \omega^2\right). \tag{16}$$

Substituting ξ_x from (7) in (3) and using (6), the longitudinal displacement of the second order is found to be

$$\xi_{1z}(2\omega) = \frac{ine^2 E^2 \exp(2i\theta)}{m^2 c\left\{\left(-4\omega^2 + \omega_p^2\right) - 2i\gamma\omega\right\}(\omega + i\gamma)} , \tag{17}$$

where $n (= kc/\omega)$ is the refractive index of the plasma due to the wave dispersion. The nonlinear polarization, electrical susceptibility and the plasma current of the second harmonic frequency, respectively are

$$P_z(2\omega) = \chi^{(2)}(2\omega) E_x(\omega) E_x(\omega) = -N_0 e\xi_{1z}(2\omega), \tag{18}$$

$$\chi^{(2)}(2\omega) = -\frac{iN_0 e^3 n}{m^2 c\left\{\left(-4\omega^2 + \omega_p^2\right) - 2i\gamma\omega\right\}(\omega + i\gamma)}, \tag{19}$$

$$j_z(2\omega) = -\frac{2N_0 e^3 n\omega E^2 \exp(2i\theta)}{m^2 c\left\{\left(-4\omega^2 + \omega_p^2\right) - 2i\gamma\omega\right\}(\omega + i\gamma)} + c.c. \tag{20}$$

Replacing ω by $\omega - i\gamma_0$ where ω and γ_0 are real frequencies and γ_0 takes account of the damping due to collisions, we find the zero frequency second order displacement $\xi_z(0)$ and the plasma current $j_z(0)$.

When $\gamma \neq \gamma_0$,

$$\xi_z(0) = \frac{ine^2 E\bar{E}\exp(-2\gamma_0 t)}{m^2 c\left\{\left(-4\omega^2 + \omega_p^2 + \gamma_0^2 - 2\gamma\gamma_0\right) + 2i\omega(4\gamma_0 - \gamma)\right\}\left\{\omega - i(\gamma_0 - \gamma)\right\}} + c.c. \tag{21}$$

When $\gamma = \gamma_0$,

eq. (21) gives

$$\xi_z(0) = \frac{ine^2 E\bar{E}\exp(-2\gamma_0 t)}{m^2 c\left\{\left(-4\omega^2 + \omega_p^2 - \gamma_0^2\right) + 6i\omega\gamma_0\right\}}\omega + c.c. \tag{22}$$

and when $\gamma \neq \gamma_0$,

$$j_z(0) = \frac{2i\, n\, e^3 \gamma_0 E N_0 \bar{E}\exp(-2\gamma_0 t)}{m^2 c\left\{\left(-4\omega^2 + \omega_p^2 + \gamma_0^2 - 2\gamma\gamma_0\right) + 2i\omega(4\gamma_0 - \gamma)\right\}\left\{\omega - i(\gamma_0 - \gamma)\right\}} + c.c. \tag{23}$$

For $\gamma = \gamma_0$,

eq. (23) gives

$$j_z(0) = \frac{ine^3 2E\bar{E}N_0\gamma_0 \exp(-2\gamma_0 t)}{m^2c\left\{\left(-4\omega^2+\omega_p^2-\gamma_0^2\right)+6i\omega\gamma_0\right\}}\omega + c.c. \quad (24)$$

These dc increments depend on the relation between the two decay constants γ_0 and γ. For a special relation between γ_0 and γ, the coefficients of the decay factor $\exp(-2\gamma_0 t)$ have a secularly growing term. But this factor grows much faster in the denominator, and so makes the secular growth in the numerator ineffective.

The continuity condition of the second order

$$\frac{\partial N_2}{\partial t} + N_0(\nabla \cdot u_2) = 0, \quad (25)$$

gives the second order fluctuation in density, because Gauss's law for the second order field, is

$$\nabla \cdot E_2 = -4\pi eN_2. \quad (26)$$

Another relation for finding this field, following from the Ampere-Maxwell law, is

$$\frac{\partial E_2}{\partial t} = -4\pi j_2. \quad (27)$$

Eq. (20) gives

$$N_2(2\omega) = -\frac{N_0 k}{\omega}u_{2z},$$

$$-\frac{2n\, e^2 k\, N_0\, E^2 \exp(2i\theta)}{m^2c\left\{\left(-4\omega^2+\omega_p^2\right)-2i\gamma\omega\right\}\{\omega+i\gamma\}} + c.c. \quad (28)$$

There is no $N_2(0)$. Eq. (26) gives

$$E_{2z}(2\omega) = -4\pi e\int N_2 dz,$$

$$-\frac{ine\omega_p^2 E^2 \exp(2i\theta)}{mc\left\{\left(-4\omega^2+\omega_p^2\right)-2i\gamma\omega\right\}\{\omega+i\gamma\}} + c.c. \quad (29)$$

which also follows from (27), on using (20). The space-independent, and non-oscillating contribution for $\gamma \neq \gamma_0$, obtained from equation (23) is

$$E_{2z}(0) = \frac{i\,n\,e\omega_p^2 E\bar{E}\exp(-2\gamma_0 t)}{mc\left\{\left(-4\omega^2+\omega_p^2+\gamma_0^2-2\gamma\gamma_0\right)+2i\omega(4\gamma_0-\gamma)\right\}\{\omega+i(\gamma_0-\gamma)\}} + c.c. \quad (30)$$

For $\gamma = \gamma_0$, eq. (24) gives

$$E_{2z}(0) = \frac{ine\omega_p^2 E\bar{E}\exp(-2\gamma_0 t)}{mc\left\{\left(-4\omega^2+\omega_p^2-\gamma_0^2\right)+6i\omega\gamma_0\right\}\omega} + c.c. \quad (31)$$

Including this $E_{2z}(0)$ in the right hand side of (29) as the constant of integration with respect to z, we find for $\gamma \neq \gamma_0$,

$$E_z(z,t) = \frac{ine\omega_p^2 E^2 \exp(2i\theta)}{mc\left\{\left(-4\omega^2+\omega_p^2\right)-2i\gamma\omega\right\}\{\omega+i\gamma\}}$$

$$\frac{ine\omega_p^2 E\bar{E}\exp(-2\gamma_0 t)}{mc\left\{\left(-4\omega^2+\omega_p^2+\gamma^2-2\gamma\gamma_0\right)+2i\omega(4\gamma_0-\gamma)\right\}\{\omega+i(\gamma_0}$$

$$+ c.c. \quad (32)$$

and when $\gamma = \gamma_0$, we find that

$$E_z(z,t) = \frac{ine^2\omega_p^2 E^2 \exp(2i\theta)}{mc\left\{\left(-4\omega^2+\omega_p^2-\gamma_0^2\right)+6i\omega\gamma_0\right\}\omega}$$

$$\frac{ine^2\omega_p^2 E\bar{E}\exp(-2\gamma_0 t)}{mc\left\{\left(-4\omega^2+\omega_p^2+\gamma_0^2\right)+6i\omega\gamma_0\right\}\omega} + c.c. \quad (33$$

The third-order increments are of transverse waves. So, thes contribute to the field-induced displacement from transvers waves. The frequency combinations of the third order bein $\omega \pm \omega \pm \omega$, the transverse displacements $\xi_x(\omega \pm \omega \pm \omega)$ ar at the frequencies ω and 3ω. So, we can write

$$\xi_x(\omega \pm \omega \pm \omega) = \xi_x(\omega) + \xi_x(3\omega). \quad (34$$

Here, $\xi_x(\omega)$ is the nonlinear evolution of the first harmoni of the field-induced displacement, correct upto the third order $\xi_x(3\omega)$, the third harmonic correct upto the third of the same is not important here. The Maxwell equations for transvers waves give

$$\ddot{E}_x - c^2 E_x'' - 4\pi e N_0 \dot{\xi}_x = 4\pi e\frac{d}{\partial t}\left(N_2\dot{\xi}_x\right). \quad (35$$

When $\gamma \neq \gamma_0$, we find that

$$\frac{e}{c}\xi_z H_y =$$

$$-\frac{2in^2e^3\left\{i\omega E^2\bar{E}\exp(i\theta)+\gamma E^2\bar{E}\exp(i\theta-3i\gamma t)\right\}}{m^3c\left\{\left(-4\omega^2+\omega_p^2\right)-2i\gamma\omega\right\}\{\omega+i\gamma\}},$$

where the dropped terms are those of $\exp(\pm 3i\theta)$. Eq. (35) g

$$\frac{e}{m}E_x = \frac{X}{n^2-1}\ \xi_x + \frac{\dot{N}_2}{N_0}\dot{\xi}_x + \frac{N_2}{N_0}\dot{\xi}_x \}, \tag{37}$$

$$\left| -\omega^2 - i\gamma\omega - \frac{\omega^2 X}{n^2-1} \right| \xi_x =$$

$$\frac{}{\left(n^2-1\right)N_0}\left(N_2\dot{\xi}_x + N_2\ddot{\xi}_x\right) + \frac{e}{mc}\dot{\xi}_z H_y, \tag{38}$$

where $X = \omega_p^2 / \omega^2$. Putting the value of ξ_x from (12) we obtain the nonlinear relation between the frequency (complex) and the wave number as

$$\left| 1 + \frac{}{n^2-1} + iz \right| =$$

$$\frac{-2n\alpha\bar{\alpha}}{(-4+X+2iz)}\left\{\frac{3X}{\left(n^2-1\right)(1-iz)} + (1-iz)\right\}, \tag{39}$$

where $\alpha = \dfrac{eE}{m\omega c}$, $\bar{\alpha} = \dfrac{e\bar{E}}{m\omega c}$, $z = \gamma/\omega$. (40)

The real and imaginary parts of (39) give the relations

$$k^2c^2 - \omega^2 - \omega_p^2 = -\frac{4\alpha\bar{\alpha}n^2\omega_p^2}{n^2+3}, \tag{41}$$

$$\gamma_0 = \frac{\gamma\omega_p^2}{2\omega^2}\left\{1 - \frac{8n^2\alpha\bar{\alpha}(n^2+2)z}{(n^2+3)^2}\right\}. \tag{42}$$

These are nonlinearly correct (upto the third order of pproximation) dispersion relations between ω and k, and the xpression for the wave damping factor γ_0. Writing

$$\omega_{pn}^2 = \omega_p^2\left(1 + \frac{4n^2\alpha\bar{\alpha}}{n^2+3}\right), \tag{43}$$

he dispersion relation becomes

$$k^2c^2 = \omega^2 - \omega_{pn}^2. \tag{44}$$

So, there is an increase in the value of the cut-off frequency due to nonlinearity.

2(c) *A simple case of dynamics of bound electrons :*

presence of an electro-magnetic wave polarized in the ZX plane, the equation of motion of the harmonically bound electron

$$\ddot{\xi} + \omega_0^2\xi + \gamma\dot{\xi} + \alpha\xi^2 = -\frac{e}{m}E_x = \frac{-e}{m}\left(E_0 e^{i\theta} + c.c.\right), \tag{45}$$

where ω_0 is the oscillator frequency and $\alpha\xi^2$ takes account phenomenologically, of the nonlinear polarization, and $\theta = kz - \omega t$. The linearized solution of this equation is

$$\xi(\omega) = \frac{eE_0\exp(i\theta)}{m\left(-\omega^2 + \omega_0^2 - i\omega\gamma\right)}. \tag{46}$$

The second harmonic correction term is

$$\xi(2\omega) = -\frac{e\alpha E^2\exp(2i\theta)}{m^2D^2(\omega)D(2\omega)}, \tag{47}$$

where

$$D(\omega) = \left(\omega_0^2 - \omega^2 - i\gamma\omega\right) = D^*(-\omega). \tag{48}$$

For two waves at frequencies ω_1 and ω_2, the solution at the beat frequency $\omega_1 - \omega_2$ is

$$\xi(\omega_1-\omega_2) = -\frac{e^2\alpha E_1E_2\exp i(\theta_1-\theta_2)}{m^2D(\omega_1)D^*(-\omega_2)D(\omega_1-\omega_2)} \tag{49}$$

where $\theta_i = k_iz - \omega_i$, $i = 1,2$. At frequency 2ω, the nonlinear polarization is

$$P^n(2\omega) = -N_0e\xi(2\omega) = \chi(2\omega,\omega,\omega)E_x^2 \tag{50}$$

where

$$\chi(2\omega,\omega,\omega) = -\frac{N_0e^2\gamma}{mD^2(\omega)D(2\omega)} \tag{51}$$

N_0 is the number density of bound electrons, $\chi(2\omega,\omega,\omega)$ is the non-linearly induced susceptibility of the second harmonic field.

If the deviation ξ of the electronic binding is of the order of the radius a of the equilibrium orbital path of the electron, the nonlinear force is of the order of the force $f_1 = -m\omega_0^2a = eE_a$ of the linearized approximation, where E_a is the atomic force ($\approx 3\times10^8$ volts/cm) binding the electron to the orbit. Therefore, the magnitude of $\xi \approx a$ and $m\omega_0^2a = m\xi^2\alpha = eE_a$ and these relations give

$$\frac{\alpha}{\omega_0^2} \approx a^{-1}. \tag{52}$$

In the ratio

$$\frac{P^n(2\omega)}{P(\omega)} \approx \frac{eE_0}{m\omega_0^2a} \approx \frac{E_0}{E_a}, \tag{53}$$

$P(\omega)\left(= N_0e\xi(\omega)\right)$ is the polarization in the linearized approximation. For the power flux density 3×10^{10} Watts/cm^2, in the focal region of a Q-switched laser, $E_0 / E_a \simeq 3 \times 10^{-3}$. So the nonlinear response is only a small perturbation, and the classical treatment is justified.

3. Dynamics of multicomponent plasma

3.(a) Basic equations :

To study the collective behavior of a multicomponent plasma, we consider that the plasma consists of free electrons, free ions, weakly bound electrons and ions and neutral atoms. The field induced average displacement of a species of particles, per unit volume at time t, from the equilibrium average position $\boldsymbol{r}$, is ξ, say. The displacement $\xi(\boldsymbol{r},t)$ should be finite and is small for waves of finite but small amplitude. So approximately,

$$\xi(\boldsymbol{r},t) = \boldsymbol{u}(\boldsymbol{r},t). \tag{54}$$

This relation is valid separately for each of the five components of plasma and we write

$$\xi = \xi_{eb} + \xi_{ef} + \xi_{if} + \xi_{ib} + \xi_n , \tag{55}$$

$$\boldsymbol{u} = \boldsymbol{u}_{eb} + \boldsymbol{u}_{ef} + \boldsymbol{u}_{if} + \boldsymbol{u}_{ib} + \boldsymbol{u}_n , \tag{56}$$

where the subscript *'eb'* stands for the bound electrons, *'ef'* for the free electrons *'ib'* for ions of bound electrons, *'if'* for ions which have released all the free electrons and *'n'* for neutral particles. $\boldsymbol{u}_{eb}$, $\boldsymbol{u}_{ib}$, $\boldsymbol{u}_{ef}$, $\boldsymbol{u}_{if}$ and $\boldsymbol{u}_n$ are, respectively, the average velocities.

The basic, self consistent system of equations of the five component fluid-like mixture constituting the plasma, are

$$\frac{\partial \boldsymbol{u}_s}{\partial t} + (\boldsymbol{u}_s \cdot \nabla)\boldsymbol{u}_s =$$

$$-\omega_{0s}^2 \xi_s - \frac{1}{\rho_s}\nabla p_s + \frac{q\ell}{m}\left[\boldsymbol{E} + \frac{1}{c}(\boldsymbol{u}_s \times \boldsymbol{H})\right] + f_s , \tag{57}$$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \boldsymbol{u}_s) = 0, \tag{58}$$

where

$$p_s = m_s C_s^2 n_s \text{ and } \rho_s = m_s n_s , \tag{59}$$

where $s = n, eb, ib, ef, if$ for neutral particles, bound electrons, bound ions, free electrons and free ions. For $l = e$, $q_l = e$, for $l = i$, $q_l = e$ and $l = n$, $q_l = 0$. For $s = eb, ef$, $m_s = m$ (mass of electrons of both type); for $s = ib, if$, $m_s = M$ (mass of ions of both types); if $s = n$ then $m_s = M_n$ (mass of the neutral particle). Again except $\omega_{0eb} (= \omega_0)$ (classical electron orbital frequency), other ω_{0s} are zero. Here p_s are the partial pressures and C_s are the acoustic speeds of all species. f_s represents collisional and other forces per unit mass acting on the five species.

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \tag{60}$$

$$\nabla \times \boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} + \frac{4\pi e}{c}\left(n_{if}\, \boldsymbol{u}_{if} - n_{ef}\, \boldsymbol{u}_{ef}\right), \tag{61}$$

$$\nabla \cdot \boldsymbol{D} = 4\pi e\left(n_{if} - n_{ef}\right), \tag{62}$$

$$\nabla \cdot \boldsymbol{B} = 0. \tag{63}$$

The electric displacement vector $\boldsymbol{D}$, and the polarization vector $\boldsymbol{P}$ which accommodate the fields of the displacement of the bound charges and their nuclei, are connected by the constitutive

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P} , \tag{64}$$

where

$$\boldsymbol{P} = e\left(n_{ib}\xi_{ib} - n_{eb}\xi_{eb}\right). \tag{65}$$

Completed use of the Lorentz theory gives the same result because then (61) is replaced by

$$\nabla \times \boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\boldsymbol{j}, \tag{66}$$

where

$$\boldsymbol{j} = \boldsymbol{j}_b + \boldsymbol{j}_f , \tag{67}$$

$$\boldsymbol{j}_b = e\left(n_{ib}\boldsymbol{u}_{ib} - n_{eb}\boldsymbol{u}_{eb}\right);\ \ \boldsymbol{j}_f = e\left(n_{if}\,\boldsymbol{u}_{if} - n_{ef}\,\boldsymbol{u}_{ef}\right. \tag{68}$$

Then the right hand side of (66)

$$= \frac{1}{c}\frac{\partial}{\partial t}\left[\boldsymbol{E} + 4\pi e\left(n_{ib}\xi_{ib} - n_{eb}\xi_{eb}\right)\right] + \frac{4\pi}{c}\boldsymbol{j}_f , \tag{69}$$

because

$$\boldsymbol{u}_{ib} = \frac{\partial \xi_{ib}}{\partial t};\ \ \boldsymbol{u}_{eb} = \frac{\partial \xi_{eb}}{\partial t}.$$

Substituting eq. (64) and (65) in (61) and (62), we get (66) and (68), and

$$\nabla \cdot \boldsymbol{E} = 4\pi(\rho_b + \rho_f), \tag{70}$$

where ρ_b and ρ_f are the perturbing charge density of the bound and free species, and $\nabla \cdot \boldsymbol{P}_b^l = -\rho_b$.

The small mixing of the phenomenological (Maxwell) and analytical (Lorentz) approaches for classical electrodynamics, obliges the use of prevalent classical concepts for the quantum effects of nonlinear polarization, through the field vectors $\boldsymbol{D}$ and $\boldsymbol{P}$ of the Ampere-Maxwell law [1].

3(b) Generalized Poynting theorem for energy flux :

A differential equation for the Poynting theorem (The law of conservation of energy of electrodynamics ; the time rate of change of electro-magnetic energy plus the energy flowing through the boundary of a certain volume equals the negative of the total work done by the medium within that volume) based on the universally valid Maxwell equations, is

$$-(E\cdot j)=\frac{\partial U_{EM}}{\partial t}+\nabla\cdot S',\tag{71}$$

$$=\frac{\partial}{\partial t}(W_E+W_M)+\nabla\cdot S',\tag{72}$$

where $U_{EM}\left[=\frac{1}{8\pi}\{(E\cdot D)+(H\cdot B)\}\right]$ is the total energy density of the field, $S'\left[=(c/4\pi)(E\times H)\right]$ is the Poynting vector, and W_E and W_M are the electric and the magnetic energy density.

We use the linearized approximation of the relations of subsection 3(a), in the collision free limit, neglecting totally f_{eb}, f_{ib}, f_{if}, f_n. So, we use (67) and (68) and obtain

$$(E\cdot j)=e\left[n_{ib}^0(E\cdot u_{ib})+n_{if}^0(E\cdot u_{if})\right]$$
$$-e\left[n_{eb}^0(E\cdot u_{eb})+n_{ef}^0(E\cdot u_{ef})\right],\tag{73}$$

where $n_{ib}^0, n_{if}^0, n_{eb}^0$ and n_{ef}^0 are number densities at equilibrium. We obtain from eq. (73), the energy loss of the wave field to the plasma, per unit volume. The equations of momentum transfer give

$$e(E\cdot u_{ib})=\frac{\partial}{\partial t}\left(\frac{1}{2}Mu_{ib}^2\right)+\frac{1}{n_{ib}^0}(u_{ib}\cdot\nabla p_{ib}),\tag{74}$$

$$e(E\cdot u_{if})=\frac{\partial}{\partial t}\left(\frac{1}{2}Mu_{if}^2\right)+\frac{1}{n_{if}^0}(u_{if}\cdot\nabla p_{if}),\tag{75}$$

$$e(E\cdot u_{eb})=-\frac{\partial}{\partial t}\left(\frac{1}{2}mu_{eb}^2+\frac{1}{2}m\omega_0^2\xi_{eb}^2\right)-\frac{1}{n_{eb}^0}(u_{eb}\cdot\nabla p_{eb}),\tag{76}$$

$$e(E\cdot u_{ef})=-\frac{\partial}{\partial t}\left(\frac{1}{2}mu_{ef}^2\right)-\frac{1}{n_{ef}^0}(u_{ef}\cdot\nabla p_{ef}).\tag{77}$$

Substituting (74) – (77) in eq. (73), we obtained the more explicit expression for the rate of work done by the applied fields in terms of the kinetic energy density of the four species of charges,

$$e(E\cdot j)=\left[\frac{\partial}{\partial t}\left(\frac{1}{2}Mn_{ib}^0u_{ib}^2+\frac{1}{2}Mn_{if}^0u_{if}^2+\frac{1}{2}mn_{eb}^0u_{eb}^2\right.\right.$$
$$\left.+\frac{1}{2}mn_{ef}^0u_{ef}^2+\frac{1}{2}mn_{eb}^0\omega_0^2\xi_{eb}^2\right)+(u_{ib}\cdot\nabla p_{ib})$$
$$\left.+(u_{if}\cdot\nabla p_{if})+(u_{eb}\cdot\nabla p_{eb})+(u_{ef}\cdot\nabla p_{ef})\right].\tag{78}$$

Their sum is

$$W_k=\left(\frac{1}{2}Mn_{ib}^0u_{ib}^2+\frac{1}{2}Mn_{if}^0u_{if}^2+\frac{1}{2}mn_{eb}^0u_{eb}^2\right.$$
$$\left.+\frac{1}{2}mn_{ef}^0u_{ef}^2+\frac{1}{2}mn_{eb}^0\omega_0^2\xi_{eb}^2\right).\tag{79}$$

Moreover, using the vector relation

$$\nabla\cdot(\rho u)=\rho\nabla\cdot u+(u\cdot\nabla)\rho,\tag{80}$$

eq. (78) becomes

$$e(E\cdot j)=\left[\frac{\partial W_k}{\partial t}+\nabla\cdot(u_{ib}p_{ib}+u_{if}p_{if}+u_{eb}p_{eb}+u_{ef}p_{ef})\right.$$
$$\left.-\left\{p_{ib}(\nabla\cdot u_{ib})+p_{if}(\nabla\cdot u_{if})+p_{eb}(\nabla\cdot u_{eb})+p_{ef}(\nabla\cdot u_{ef})\right\}\right].\tag{81}$$

With the help of linearized approximation of the continuity equations (58) of mass, this equation can be written as

$$e(E\cdot j)=\frac{\partial}{\partial t}(W_k+W_p)+\nabla\cdot S',\tag{82}$$

where

$$\frac{\partial W_p}{\partial t}=\left(\frac{p_{ib}}{n_{ib}^0}\frac{\partial n_{ib}}{\partial t}+\frac{p_{if}}{n_{if}^0}\frac{\partial n_{if}}{\partial t}+\frac{p_{eb}}{n_{eb}^0}\frac{\partial n_{eb}}{\partial t}+\frac{p_{ef}}{n_{ef}^0}\frac{\partial n_{ef}}{\partial t}\right);\tag{83}$$

when $n_{ib}^0=n_{eb}^0=n_b^0$ and $n_{if}^0=n_{ef}^0=n_f^0$, then

$$=\frac{\partial}{\partial t}\left\{\frac{(p_{ib}n_{ib}+p_{eb}n_{eb})}{2n_b^0}+\frac{(p_{if}n_{if}+p_{ef}n_{ef})}{2n_f^0}\right\}\tag{84}$$

and

$$S'=(p_{ib}u_{ib}+p_{ib}u_{if}+p_{eb}u_{eb}+p_{ef}u_{ef}).\tag{85}$$

Here, S' is the energy flux density carried away by the plasma constituents due to the pressure force ; and W_p is the potential energy density, stored due to compressibility in the plasma. Now, equating the expressions for $(E\cdot j)$ in (72) and (82), we obtain the energy conservation law

$$\frac{\partial W}{\partial t}+\nabla\cdot S=0,\tag{86}$$

where

$$W = W_K + W_P + W_E + W_M\,;\quad S = S' + S^l\,. \tag{87}$$

Ratio of the kinetic energy density of bound and free electrons is

$$\frac{W_{kb}}{W_{kf}} = \frac{n_b^0 u_{ib}^2 + (m/M) n_b^0 u_{eb}^2}{n_f^0 u_{if}^2 + (m/M) n_f^0 u_{ef}^2} + \frac{n_b^0 \omega_0^2 \xi_{eb}^2}{(M/m) n_f^0 u_{if}^2 + n_f^0 u_{ef}^2}\,. \tag{88}$$

(a) Since $m/M \ll 1$, $u_{eb} \approx u_{ib}$ and $u_{ef}^2 > u_{if}^2$, the second term is small; so neglecting that, we get

$$\frac{W_{kb}}{W_{kf}} = \frac{n_b^0 u_{ib}^2}{n_f^0 u_{if}^2}\,. \tag{89}$$

If $u_{if}^2 >> u_{ib}^2$, and $n_b^0 < n_f^0$ eq. (89) gives $W_{kb} < W_{kf}$.

(b) When $u_{ef}^2 >> u_{if}^2$, $(m/M) << 1$ and $u_{eb} = u_{ib}$, neglecting the second term in (88) we find that

$$\frac{W_{kb}}{W_{kf}} = \frac{n_b^0 u_{ib}^2}{n_f^0 u_{if}^2 + (m/M) n_f^0 u_{ef}^2}\,. \tag{90}$$

When the number density of bound electrons is small $\left(n_b^0 << n_f^0\right)$, we find that $(W_{kb}/W_{kf}) << 1$. The field induced kinetic energy density of the system is less in presence of bound electrons. But the bound electrons have rotational energy from the centrifugal force.

If $\boldsymbol{F}$ is the vector sum of all the forece and $\boldsymbol{u}$ is the vector sum of the average velocity of all the species, the system of eqs. (57) – (63) give

$$\boldsymbol{F} = \left(\sum_i n_i^0 q_i\right)\boldsymbol{E} + \frac{1}{c}\left(\sum_i n_i^0 q_i \boldsymbol{u}_i\right)\times \boldsymbol{H} - m n_b^0 \omega_0^2 \xi_{eb} + \sum_i f_i\,, \tag{91}$$

$$\boldsymbol{u} = \sum u_i = \boldsymbol{u}_{eb} + \boldsymbol{u}_{ef} + \boldsymbol{u}_{ib} + \boldsymbol{u}_{if} + \boldsymbol{u}_n \tag{92}$$

and $(\boldsymbol{F}.\boldsymbol{u})$ is the rate of work done by the forces on the medium per unit volume at a point. The electro magnetic rate of work done, $(\boldsymbol{E}.\ \boldsymbol{j})$, follows from the scalar product of the first two terms of this $\boldsymbol{F}$, both containing the summation sign Σ_i, with the first terms of this $\boldsymbol{u}$. So, a broader analysis of the rate of work done is possible on the basis of the product $(\boldsymbol{F}.\boldsymbol{u})$.

3.(c) The Maxwell stress elements :

In continuum mechanics, including the field theory of electro dynamics, the expression for the force per unit volume $\boldsymbol{F}$ at a point in the space-time continuum, can be written with the help of the relevant field equations as the sum of the time derivative of one quantity $\boldsymbol{G}$, called the momentum density, and the directional derivatives with respect to the space coordinates of other, p_{ij}, say, which are called the stress elements, or the components of the pressure tensor.

The momentum transfer equations (57), neglecting $f_{ef}, f_{eb}, f_{if}, f_{ib}, f_n$ show that the total force is

$$\boldsymbol{F} = -m n_{eb}^0 \omega_0^2 \xi_{eb} - \nabla p + \rho_0 \boldsymbol{E} + \frac{1}{c}(j \times \boldsymbol{B})\,, \tag{93}$$

where

$$p = p_{eb} + p_{ef} + p_{ib} + p_{if} + p_n\,, \tag{94}$$

$$j = e\left(n_{if}^0 u_{if} - n_{ef}^0 u_{ef}\right), \tag{95}$$

$$\rho = e\left(n_{if}^0 - n_{ef}^0\right). \tag{96}$$

Using continuity equation for n_{eb} in the first term, (62) in the third term and (61) in the fourth term, (93) can be written as

$$\boldsymbol{F} = -\nabla p + \sum_{i,j} Xi \frac{\partial}{\partial x}\left(m n_{eb}^0\, \omega_0^2 \xi_{ebi}\, \xi_{ebj}\right)$$

$$+ m n_{eb}^0 \omega_0^2\, (\xi_{eb}\cdot \nabla)\xi_{eb} + \frac{1}{4\pi}\boldsymbol{E}(\boldsymbol{E}\cdot \boldsymbol{D})$$

$$+ \frac{1}{4\pi}(\nabla \times \boldsymbol{H})\boldsymbol{H} - \frac{1}{4\pi c}\left(\frac{\partial \boldsymbol{D}}{\partial t}\times \boldsymbol{H}\right), \tag{97}$$

where $\boldsymbol{X}_i$ is the unit vector parallel to the i-th space direction the coordinate system employed. Next using (64), it is written a

$$\boldsymbol{F} = -\sum_{i,j} Xi \frac{\partial}{\partial x} p_{ij} + m\omega_0^2 n_{eb}^0 (\xi_{eb}\cdot \nabla)\xi_{eb} + \boldsymbol{E}(\nabla\ \boldsymbol{P})$$

$$\cdot \frac{1}{c}\left(\frac{\partial \boldsymbol{P}}{\partial t}\times \boldsymbol{E}\right) - \frac{\partial \boldsymbol{G}}{\partial t}\,, \tag{98}$$

where

$$\boldsymbol{G} = \frac{1}{4\pi c}(\boldsymbol{E}\times \boldsymbol{H}),$$

$$T_{ij} = \frac{1}{4\pi}\left(E_i E_j + H_i H_j\right) - \frac{1}{8\pi}\left(E^2 + H^2\right)\delta_{ij}\,.$$

$$p_{ij} = p\delta_{ij} + m\omega_0^2 n_{eb}^0 \xi_{ebi}\xi_{ebj} - T_{ij}\,.$$

$$\delta_{ij} = 1 \text{ if } i = j \text{ and} = 0 \text{ if } i \neq j\,.$$

$\boldsymbol{G}$ is the electro magnetic momentum per unit volume, is the ij-th component of the Maxwell stress tensor, both matter free space; δ_{ij} is the kroneker delta. Here, the vector calculus formula

$$\frac{1}{4\pi}E(\nabla\cdot E)+\frac{1}{4\pi}(\nabla\times H)\times H-\frac{1}{4\pi c}\left(\frac{\partial E}{\partial t}\times H\right.$$

$$=-\frac{\partial G}{\partial t}+\sum_{ij} Xi\frac{\partial T_{ij}}{\partial x_j} \tag{103}$$

and Faraday's law of induction have been used, since

$$E(\nabla\times P)=\nabla\times(E\times P)-(P\cdot\nabla)E+(E\cdot\nabla)P+P(\nabla\cdot E), \tag{104}$$

$$\nabla\times(E\times P)=\sum Xi(E\times P)_k\,\varepsilon_{ijk}\ . \tag{105}$$

where ε_{ijk} is the Levi civita symbol, we can include the term of (105) within p_{ij}. And since P depends on the field induced material displacement, it is determined in terms of the incident field and the material parameters. So, further analysis of F for finding the stress elements and their character, requires the knowledge of specific cases of interaction of waves with the plasma

4. Concluding remarks

We have neglected the gravitational force field of the plasma constituents and the collisional loss effects. So for simplicity, the equations for the dynamics of the neutral particles are not connected with those of the charged species. The collisional loss force per unit volume is the sum term $-\sum_j \gamma_{ij} M_i n_i(u_i-u_j)$ in the momentum transfer equation of the i-th species fluid. The expression for the frequency of collisions of the bound electrons with members of their own species and with the other species of particles ν_{hj} (say), is to be determined by the atomic theory of collisions, and is beyond our present scope. For practical purposes, the empirical relation $\nu_{hj}=\delta\nu_{ej}$ can find ν_{hj} in term of the collision frequency ν_{ej} of the free electrons with particles of their own species and with particles of the other species, where the adjustable loss factor δ is to be determined experimentally.

Using the theory for collective effects of bound electrons, free electrons and ions in the plasma, the propagation of transverse wave in a Vlasov plasma have been investigated by Chakraborty *et al* [8]. They have obtained the expression for Lagrangian and Hamiltonian density in the plasma considering the effects of bound electrons. From the expression of Thomson scattering, it is found that the scattering cross section is dependent on the Rayleigh scattering susceptibility. Moreover, the wave-plasma interaction in a plasma considering the collective effects of bound electrons, free electrons and ions may be theoretically investigated for knowing the magnetic moment field in the resonant case which would be studied in our next work.

Collective effects of a plasma containing dusty plasma, and vortex linked negatively charged heavy elements should be similarly considered. Collision frequencies of these species, for practical purposes, may be determined empirically according to the suggested rule for bound electrons. It will be interesting to know the physics of the breamsstrahlung and the inverse breamsstrahlung due to the existence of these species in a plasma.

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